Robust Diffusion Tensor Estimation by Maximizing Rician Likelihood

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INTRODUCTION

- Diffusion tensor imaging (DTI) is widely used to characterize white matter in health and disease.
- Prevalent tensor estimation methods tend to approximate the noise structure in DTI [1,4,5], yet direct consideration of Rician distributions has been applied to reduce bias and improve reliability for traditional MRI methods [cf., 2,3].
- DTI methods have not taken into account (1) the propagation of Rician noise into the tensor domain or (2) the dependence between observed attenuations caused by the use of common reference scans.
- These systematic differences can cause quantities derived from tensors — e.g., fractional anisotropy and apparent diffusion coefficient — to diverge from their true values, potentially leading to artifactual changes that confound clinically significant ones.
- Recent developments with Diffusion Tensor Estimation by Maximizing Rician Likelihood (DTEMRL) show that tensor estimation can be performed by considering the joint distribution of all observed data in the context of an augmented tensor model that accounts for Rician noise [6].

We present a robust extension of maximum likelihood tensor estimation (rDTEMRL) to improve reliability in low SNR and artifact prone applications.

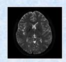
METHOD AND THEORY

Stejskal-Tanner Equation

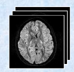
$$\frac{S_{\mathrm{dw}}}{S_{\mathrm{ref}}} = e^{-b\mathbf{g}^T \mathbf{D}\,\mathbf{g}}$$

Observed Data

Minimally Weighted Reference (ref)



Diffusion Weighted (dw)



Definition of the Rician Likelihood Model (8 parameters) [6]

$$S_{\mathrm{ref}} \sim \mathcal{R}(S_{\mathrm{ref}_0}, \sigma_{xy}) \qquad S_{\mathrm{dw}} \sim \mathcal{R}(S_{\mathrm{ref}_0} e^{-b\mathbf{g}^T \mathbf{D}\,\mathbf{g}}, \sigma_{xy})$$

$$\mathcal{L}(S_{\mathrm{dw}_1} \ldots S_{\mathrm{dw}_n}, S_{\mathrm{ref}}) = \mathcal{R}(S_{\mathrm{ref}}, S_{\mathrm{ref}_0}, \sigma_{xy}) \prod_{i=1}^{N} \mathcal{R}(S_{\mathrm{dw}_i}, S_{\mathrm{ref}_0} e^{-b\mathbf{g}_i^T \mathbf{D}\,\mathbf{g}_i}, \sigma_{xy})$$

TENSOR PARAMETERIZATION

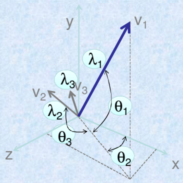
Traditional Approach

$$\mathrm{D} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{bmatrix}$$

$$= \mathrm{R}^T \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix} \mathrm{R}$$

$$= \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix} \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^T$$

Euler Angles and Eigenvalues

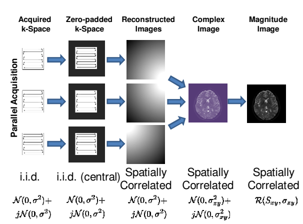


Proposed Approach

$$= \mathrm{R}_{\mathrm{abc}} \begin{bmatrix} e^{l_1} & & \\ & e^{l_2} & \\ & & e^{l_3} \end{bmatrix} \mathrm{R}_{\mathrm{abc}}^T$$

- Rodriguez parameters based on quaternions
- Eigenvalue Logarithms

NOISE ESTIMATION



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FORMULATION OF TENSOR ESTIMATION

$$\hat{\mathrm{D}} = \underset{\mathrm{D}, S_{\mathrm{ref}_0}, \sigma_{xy}}{\operatorname{argmax}}\ \mathcal{L}(S_{\mathrm{dw}_1} \ldots S_{\mathrm{dw}_n}, S_{\mathrm{ref}})$$

$$\hat{\mathrm{D}} = \underset{\mathrm{D}, S_{\mathrm{ref}}, \sigma_{xy}}{\operatorname{argmax}}\ \mathcal{L}(S_{\mathrm{dw}_1} \ldots S_{\mathrm{dw}_n}, S_{\mathrm{ref}}) P(\sigma_{xy})$$

$$\hat{\mathrm{D}} = \underset{\mathrm{D}, S_{\mathrm{ref}}, \sigma_{xy}}{\operatorname{argmax}}\ \mathcal{L}^*(S_{\mathrm{dw}_1} \ldots S_{\mathrm{dw}_n}, S_{\mathrm{ref}}) P(\sigma_{xy})$$

Robust Likelihood

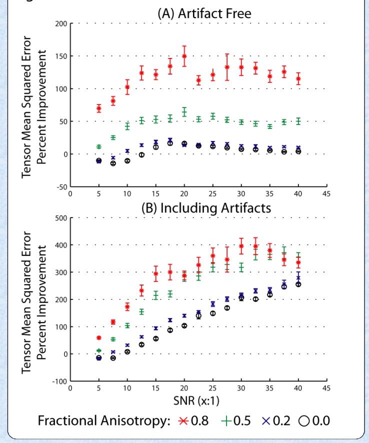
- Gaussian priors on the noise level regularize the estimated noise field with a Bayesian *a posteriori* approach. The mean of the prior is set to the estimated noise level while the standard deviation of the prior is proportional to the square of the SNR, so that the impact of the prior diminishes at high SNR.
- Huberized (truncated) likelihood measures reduce the impact of artifacts on tensor estimation. The truncation point is determined adaptively from the likelihood distribution, and set to exclude observations that fall outside two standard deviations from the median likelihood.

EXPERIMENTS AND RESULTS

Simulations

- Simulation experiments were performed with prolate tensors (i.e., tensors with identical second and third eigenvalues).
- The maximum (parallel) diffusivity was set to 2×10^{-3} mm²/s and the radial diffusivities were adjusted to create tensors with fractional anisotropies of 0, 0.2, 0.5, and 0.8.
- Artifacts were simulated by randomly attenuating one diffusion weighted (DW) image by a factor of 10. Simulated DTI studies were conducted at a b-value of 1000 s/mm² with 30 DW and 5 non-averaged reference images.
- Simulations indicated improved performance of rDTEMRL over LLMMSE in the absence of artifact (**Fig. 1A**) as low as 10:1 for gray matter (GM) like voxels (FA≤0.2) and 5:1 for white matter voxels (FA>0.2).
- In the presence of artifact, simulations demonstrate improvements in mean squared error across all noise levels studied (**Fig. 1B**).

Fig 1. Simulation: Relative Tensor Estimation Errors



In Vivo

- 22 repeated in vivo scans (acquired over 3 days) of a control subject (male, 24 y/o) were individually analyzed with both LLMMSE and rDTERML.
- Briefly, the data were acquired with a spin echo EPI sequence (TR/TE=3632/100, 0.9375 in plane, 2.5 mm slice thickness) on a 1.5T system (Intera, Philips Medical Systems, Best, The Netherlands).
- *In vivo* data (**Fig. 2**) show substantially improved performance in the major WM tracts.
- There was a mean improvement of 15% in FA in white matter tracts (FA>0.2)
- The degree of reliability improvement can be appreciated from **Fig. 3** (showing twenty two tensors estimated at a single location within the WM).

Fig 2. Tensor Variability LLMSE vs rDTEMRL

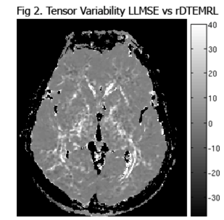
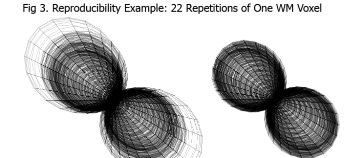


Fig 3. Reproducibility Example: 22 Repetitions of One WM Voxel



Conclusion

- Despite the "near-Gaussian" distributions of DTI experimental data, DTEMRL offers substantial improvements in estimation of tensor coefficients in WM. Simulations demonstrate consistent and significant improvements with low clinical SNR to high SNR acquisitions.

DISCUSSION

- rDTEMRL considerably improves the reliability of diffusion tensor estimates over the traditional methodology by exploiting the Rician noise distributions of MR data. The method is robust to low SNR and substantial artifact.
- As with DTEMRL, rDTEMRL does not use any spatial regularization in the estimation process, and it is possible that spatial regularization could further improve these results. An important question for further study is how rDTEMRL relates to other robust tensor estimation methods; understanding the relative benefits and tradeoffs between M-estimator methods could lead to improved robustness and performance over a wider range of SNR.

Robust likelihood based estimation of diffusion tensor reduces the impacts of artifacts and improves reliability.

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