Optimal Partitioning of Diffusion Weighting Schemes

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Introduction

A central consideration in diffusion tensor imaging is the choice of a diffusion weighting directions (DWD) along which to apply encoding magnetic fields gradients. While minimum of six independent DWD and one minimally weighted (b0) scan are required to estimate a six degree of freedom tensor model, many more are often acquired to improve the SNR and reduce bias. Optimization criteria for selecting DWD include potential energy (PE) [1], condition number [2], and rotational invariance of the condition number [3]. We address situations in which it is desirable to be able to use subsets of the available data to compute tensor, such as empirically estimating bounds on observation variability, exploring of the comparative benefits of different DWD sets, or comparing new acquisition schemes to legacy ones. Existing optimization methods do not consider the optimality of partitions, so small perturbations from the global optimum could result in greatly improved optimality of particular partitions. We present novel methods to derive optimal subsets from an existing DWD set and to joint optimize a set and subsets of particular sizes.

Methods

Subset Selection: There are a combinatorial number of possible partitions and an exhaustive search of this space becomes problematic. We apply a Monte Carlo (MC), pair wise relaxation algorithm to arrive at locally optimal partitions. For each random initialization, sequential pair-wise exchange of status is explored until no pair-wise exchange results in lower potential energy.

Joint Optimization: We extend the PE criteria to be the weighted sum of the PE criteria of the desired set and subset(s), and employ a MC relaxation algorithm to numerically find locally optimal sets. We initialize with a random set of N points on the surface of the unit sphere. We address the optimality criteria as the problem of positioning maximally distributed points (by minimum electric potential) with the constraint that the set must contain both the point and its reflection about the origin. A pseudo-force equal to the weighted sum of the forces that would be present given the set and each partition is used to drive relaxation. Once the PE improvement over an iteration drops below a threshold, a local minimum is declared.

Results and Discussion

We compare our partitioning algorithms' results to the optimal DWD sets (based on PE) presented in Table 1.1. Optimal subsets selected from the optimal 30 (highlighted) are similar to the optimal sets of the corresponding size (Table 1.2). Joint optimization of sets and subsets introduces nominal changes in the optimality of the 30 DWD set (Table 1.3), while improving optimality criteria of the subsets (Table 1.4), especially in terms of minimum angular separation and maximum condition number. Visually, the differences in optimality criteria can be appreciated form the spherical Voronoi tessellations at right.

The subset selection method reliably achieves close approximations of optimal sets of smaller size than the original provided that such sets exist. Regularity of spacing and density of placement improve the partitioning success. The joint optimization method is appropriate for working with any number of DWD, and the degree of trade off between optimally of the entire DWD set and that of the subsets can be adjusted to suit a particular study by adapting the weights during optimization. These methods enable efficient use and reuse of diffusion weighted data. This approach can be readily extended to derived optimal extensions of existing DWD sets such that resulting set had a subset exactly equivalent to a specified set. In summary, these methods enable comparison of results from multiple DWD schemes using a single acquisition.

References

[1] Jones, et al. (1999), MRM 42():515, [2] Skare, et al. (2000), JMR 147(2):340.
[3] Batchelor et al. (2003) MRM 49(6):1143

Traditional PE Optimization

1.1 Optimal PE DWD

#	PE	Min Ang[1]	Cond # (min-max)[2]
30*	3087.7	25.6°	1.59 (1.58-1.59)
15	719.5	37.0°	1.60 (1.59-1.60)
10	301.8	46.0°	1.64 (1.55-1.67)
6*	98.3	63.4°	1.58 (1.58-1.58)

1.2 Best Subset of Optimal 30 DWD

#	PE	Min Ang	Cond # (min-max)
-	-	-	-
15	729.1	27.5°	1.60 (1.57-1.89)
10	303.5	31.2°	1.61 (1.59-1.81)
6*	99.0	51.2°	2.10 (1.78-2.28)

Joint Optimization of PE for 30 DW and Subset

1.3 Joint Optimized 30 DWD

#	PE	Min Ang	Cond # (min-max)
30	3089.9	24.4°	1.59 (1.58-1.61)
30	3088.0	24.9°	1.58 (1.58-1.59)
30*	3087.7	25.7°	1.58 (1.58-1.59)

1.4 Joint Optimized Subset DWD

#	PE	Min Ang	Cond # (min-max)
15	722.5	30.6°	1.62 (1.58-1.67)
10	302.5	40.9°	1.64 (1.59-1.79)
6*	98.6	60.2°	1.84 (1.65-1.96)

[1]Minimum Angular Separation. [2]Condition Number. * Spherical Voronoi diagram of DW set shown.