REORIENTATION STRATEGIES FOR HIGH ORDER TENSORS

F. Renard1,2, V. Noble1, C. Heinrich1, S. Kremer2

1 LSIIT, UMR UDS-CNRS 7005, Bld S. Brant BP 10413, 67412 Illkirch Cedex, France
2 LINC, FRE UDS-CNRS 3289, 4 rue Kirschleger, 67085 Strasbourg Cedex, France
felix.renard@lsiit.u-strasbg.fr

ABSTRACT

In Diffusion Magnetic Resonance Imaging, the diffusion process is commonly modeled by a symmetric 2nd order tensor. Since 2nd order tensors are not able to properly model crossing fibers, High Order Tensors (HOTs) can be considered in particular to model the Orientation Distribution Function. Applying a spatial transformation to such tensor fields requires an appropriate reorientation strategy to preserve the relevancy of the orientational information related to fiber organization. We propose a general framework for HOT reorientation, relying on the decomposition of an HOT as a function of several 2nd order tensors. Contrary to existing methods, this approach enables to reorient fibers from a crossing independently. Experiments on simulated data highlight the benefit of the proposed method.

Index Terms—DTI, HOT, registration, spatial normalization, reorientation

1. INTRODUCTION

Diffusion Magnetic Resonance Imaging (D-MRI) is a recent MRI modality [1], which permits to probe local diffusion properties and thus to characterize the underlying neuronal structures. The diffusion process is commonly modeled by a symmetric 2nd order tensor called Diffusion Tensor (DT). Applying a spatial transformation to that kind of images requires to consider an appropriate reorientation strategy to preserve the relevancy of the orientational information related to fiber organization [2]. In [2], Alexander et al. compared three reorientation strategies: no reorientation, the Finite Strain (FS) and the Preservation of the Principal Direction (PPD) strategies. The last one has been shown to be the most effective and is commonly used to warp 2nd order tensor images.

Nevertheless, 2nd order tensors are not able to model complex neuronal structures such as fiber crossings. In [3], Orzaslan et al. proposed to model the diffusion process using High Order Tensors (HOT), which are able to deal with fiber crossings. To the best of our knowledge, only Barmpoutis et al. addressed the problem of HOT images registration and proposed an appropriate reorientation strategy for diffusion functions [4], [5]. In their work [5], they estimated a rotation transformation and applied it to each HOT. In the case of fiber crossing, it appears to be insufficient since it avoids the possibility to reorient fibers independently. In this paper, we propose an original reorientation strategy which corrects the orientation of each fiber of the crossing separately. Since the diffusion function is not maximal in the direction of the fiber in the case of fiber crossing, we choose to consider the Orientation Distribution Function (ODF) which describes directly the orientation of fibers and which can be efficiently be modeled using Spherical Harmonics (SHs) [6],[7]. Since there is an analytical relationship between SH and HOT parameters [7], the ODF can also be modeled by HOTs. In the rest of the paper, we assume that the ODF is described by HOTs, and more particularly, we will restrict our study to the case of 4th order tensors.

For the proposed reorientation strategy, the main idea is that an HOT modeling an ODF can be decomposed as a function of several 2nd order tensors. Two decompositions have been considered (see [8] and [9]). These decompositions are assumed to reflect the neuronal structures: each fiber of a crossing is represented by at least one 2nd order tensor. Each of the 2nd order tensors of these decompositions is reoriented with a slightly modified version of the PPD strategy to finally obtain the reoriented HOT. In the second section, we present the state of the art dedicated to the reorientation of the diffusivity function for both DT and HOT, then we introduce the two considered HOT decompositions and the proposed reorientation strategy in the case of ODF. Finally, we present results that investigate the influence of the decomposition in the proposed reorientation strategy and that compare the proposed framework with two HOT reorientation strategies from the literature.

2. STATE OF THE ART OF REORIENTATION

The reorientation strategies will be described in the context of affine image warping. However, these methods can be generalized to any kind of transformation by assuming the transformation to be locally affine via the calculus of its Jacobian matrix [2].
2.1. Reorientation strategies for 2nd order tensors

The first naive idea, considered in [2], is not to reorient the tensors. This strategy may be satisfactory for small deformations but it fails to preserve the global structure of fiber organization for larger deformations.

The other strategies proposed for 2nd order tensors are based on the estimation of a rotation transformation, thus avoiding to modify the shape of tensors (diffusion properties such as mean diffusivity and fractional anisotropy are preserved), and permitting to keep the underlying neuronal structures within each voxel.

The FS strategy consists in decomposing an affine transformation $F$ into a rotation component $R$ and a deformation component $U$, such as $F = RU$. $R$ is estimated as $R = (FF^T)^{-1/2}F$, then applied on each DT. The main drawback of this method is that the reorientation strategy does not take care of the structure of the tensor and specially of its orientation. Some transformations, such as shearing, stretching or non uniform scaling may require a reorientation that depends on the orientation of the original structure [2].

The PPD strategy tackles this problem. It consists in estimating a rotation that aligns the first eigenvector $e_1$ with $Fe_1$ and that maps the second eigenvector $e_2$ in the plane defined by $Fe_1$ and $Fe_2$. Thus, the eigenvectors $n_i$ of the reoriented tensor are given by:

$$ n_1 = \frac{Fe_1}{||Fe_1||}, n_2 = \frac{Fe_2 - (n_1^T Fe_2)n_1}{||Fe_2 - (n_1^T Fe_2)n_1||}, n_3 = n_1 \times n_2. \quad (1) $$

2.2. Reorientation strategies for HOTs

The first naive idea is also not to reorientate the HOTs. As said before, this strategy fails to preserve the global structure of fiber organization in the case of large deformations.

When modeling the diffusion process with HOT, the diffusivity function $d(g)$ can be written as:

$$ d(g) = \sum_{i+j+k=4} D_{i,j,k} g_1^i g_2^j g_3^k, \quad (2) $$

where $g = [g_1, g_2, g_3]^T$ is the magnetic field gradient direction and $D_{i,j,k}$ are the coefficients of the HOT. The expression of the reoriented diffusivity function $d_{reo}$ obtained after applying a transformation $T$ (a rigid transformation has been considered in [5] and an affine transformation in [4]) writes:

$$ d_{reo}(g) = \sum_{i+j+k=4} D_{i,j,k}(T_1 g)^i (T_2 g)^j (T_3 g)^k, \quad (3) $$

where $T_i$ corresponds to the $i$th row of the matrix transformation $T$. From Eq.3 and Eq.2, we may derive a relation directly on the HOT such as: $D_{reo} = C(T).D$, where $D$ and $D_{reo}$ are 15x1 vectors containing the coefficient $D_{i,j,k}$ of the initial HOT and of the reoriented HOT respectively, and $C$ is a 15x15 matrix, whose coefficients are derived from the parameters of the transformation $T$ [4].

When the transformation $T$ is affine [4], the shape represented by the HOT may be strongly altered for large deformations. This kind of method does not preserve the integrity of the warped cerebral matter, i.e. a voxel characterized by an isotropic diffusion can become anisotropic in the warped space. We refer to it as the basic reorientation (BR).

When the transformation $T$ is rigid [5], HOT’s shape is not altered since the tensor is merely subject to a rotation. We can assimilate this strategy to the FS strategy of the DT [2]. The integrity of the structure is preserved but the fibers cannot be reoriented independently within a fiber crossing. Thus, the non-rigid part of the transformation is unfortunately not taken into account using this reorientation.

3. METHODS

3.1. HOT decompositions

We consider the following two methods that enable to decompose an HOT as a function of 2nd order tensors. In [8], it has been shown that a fourth order tensor $D$, represented by a 6x6 matrix using the Voigt notation, can be decomposed in six 2nd order eigentensors $D_i = [D_{i,xx}, D_{i,yy}, D_{i,zz}, D_{i,xy}, D_{i,xz}, D_{i,yz}]$ such as:

$$ D = \sum_{i=1}^{6} \lambda_i D_i D_i^T, \quad (4) $$

where $\lambda_i$ are the corresponding eigenvalues. The six eigentensors are not necessarily positive definite. We refer to this decomposition as the spectral decomposition (SD). In [8], the HOT models a covariance, but there is no restriction to consider the spectral decomposition in our application, since HOTs used to model the fibers orientation have the same properties of symmetry and positivity.

In [9], fourth order tensors are expressed as ternary quartic forms. Thanks to the Hilbert’s theorem, ternary quartics can be rewritten as follows:

$$ d(g) = (v^T D_1)^2 + (v^T D_2)^2 + (v^T D_3)^2, \quad (5) $$

where $g = [g_1, g_2, g_3]^T$ is the position on the unit sphere, $v = [g_1^2, g_2^2, g_3^2, g_1 g_2, g_1 g_3, g_2 g_3]$, and $D_i = [D_{i,xx}, D_{i,yy}, D_{i,zz}, D_{i,xy}, D_{i,xz}, D_{i,yz}]$, corresponding to the quadratic form $i$. The authors of [9] proposed to use the Iwasawa decomposition via a least squares method to estimate the coefficients of the three quadratic forms $D_i$. We refer to this decomposition as the Hilbert’s decomposition (HD).

3.2. The proposed HOT reorientation strategy

In this section, we assume that, under a transformation $F$, the underlying fibers modeled by an HOT can be reoriented independently, and the diffusion properties of each fiber must be preserved. We also assume that the decompositions proposed
in [8] and [9] can represent naturally the underlying neuronal structures, *i.e.* the 2nd order tensors that have a prominent influence in the decomposition are representative of the main fiber bundles.

For the proposed original reorientation strategy, we consider the decompositions presented in 3.1 and then apply the PPD algorithm on each 2nd order tensor $D^i$ obtained by these decompositions. The PPD has been designed for reorienting symmetric positive-definite second order diffusion tensors [2]. Unfortunately, the decompositions do not guarantee the $D^i$ to be positive definite. We have to adapt the PPD algorithms to cope with this limitation. We can notice that the two decompositions involve the square of $D^i$. Consequently, we consider as the principal eigenvector of $D^i$ for the PPD the one with the maximum absolute value of its eigenvalues. The last step is to reconstruct the HOT from its reoriented decomposition. The calculation is straightforward for the SD via Eq.4. For the HD, the equations to derive HOT coefficients from the parameters of the decomposition can be found in [9]. We refer in the sequel as SD-PPD for the strategy using the spectral decomposition, and HD-PPD for the second one using the Hilbert’s theorem.

4. EXPERIMENTAL RESULTS

The first experiment consists in verifying visually the main assumption on which the proposed method is based, namely that 2nd order tensors obtained by decompositions presented in 3.1 correctly fit the underlying fibers. In Fig.1, an HOT model representative of an orthogonal 2-fiber crossing (Fig. 1.a) and the contribution of the two prominent tensors in the corresponding HD decomposition (Fig. 1.b) are plotted. In this case, the SD and the HD lead to similar decompositions with respect to the two prominent tensors obtained.

To evaluate the performances of the HD-PPD and SD-PPD, we compare them with the BR and FS strategies applied here for ODF reorientation. To this end, we synthesized an ODF field depicting a 2-fiber crossing. We applied a spatial non linear transformation (a sinus function), and then HOTs are reoriented using the different strategies. Finally, we re-sorted to a linear interpolation between HOT coefficients in order to resample the data on a regular grid. To evaluate the efficiency of the reorientation strategies, we have also plotted the Generalized Anisotropy (GA) [3] representing the degree of anisotropy of an HOT model. The results are displayed in Fig. 2. An efficient reorientation strategy should fulfill the two following properties:

- preservation of the local diffusion properties;
- preservation of the global underlying structures (fibers).

The first property can be observed in the second column of Fig. 2, and the second property in the first one.

We can observe (Fig. 2, 2nd row), that the Basic Reorientation does not preserve the integrity of the neuronal structures. The Finite Strain method seems to preserve the local diffusion properties (Fig. 2, 3rd row). Nevertheless, as suggested in [2], this method does not take into account the whole information of the transformation. We can observe that this method cannot well reorient HOTs in the fiber crossing since it does not allow the two fibers to be reoriented separately (the vertical fiber should have been left unaffected by the transformation). The PPD-SD can preserve the global structure of the fibers (Fig. 2, 4th row). Nevertheless, diffusion properties in isotropic regions are altered. We can explain this by the fact that an HOT representing a sphere is decomposed in six 2nd eigentensors associated with similar weights, but unfortunately, the transformation has not the same impact on each eigentensor since their orientations are different thus possibly leading to anisotropic ODF. The PPD-HD appears to better preserve the local and global structures (Fig. 2, 5th row). The main difference between the PPD-HD and the PPD-SD is the number of second order tensors in the decomposition. Moreover, in isotropic regions, the HD leads to the decomposition of a sphere in one prominent tensor and two other tensors associated with low eigenvalues, thus avoiding the pitfalls encountered with the SD decomposition.

The influence of the noise on the two decompositions and the consequences on the reorientation are also investigated. To this end, we first reorient an initial fourth order tensor $D_{init}$, yielding $D_{reco}$ with the different reorientation strategies. Then, we add Gaussian white noise on each coefficient of $D_{init}$ and reorient it in $D_{noisy_{reco}}$. This process is repeated $N$ times to estimate the variance of the noise on the reoriented tensor, as:

$$error = \frac{1}{N} \sum_{i=1}^{N} ||D_{reco} - D_{noisy_{reco}}||^2,$$

where $||.||$ is the Euclidean norm. $N$ has been set to 10 000 in our experiments. Adding Gaussian white noise on each coefficient of $D_{init}$ may not be a realistic model, but it is sufficient to highlight the robustness of the two decompositions. Notice that the FS and BR strategies are not sensitive to the noise that corrupts the data since the reorientation is estimated only by considering the information from the transformation. The results are presented in Table 1 for various levels of noise. The two decompositions do not seem to be so much affected by the noise since the variance of the noise in the reoriented HOTs is similar to the variance of the noise simulated in the initial HOTs (see Table 1).

The last but the not the least important point of comparison between the methods is the computation time (see table
order tensors. Two decompositions have been investigated: the spectral and the Hilbert’s decompositions. The HD-PPD strategy appears to be the best performer since it seems to better preserve the underlying white matter structures. Nevertheless, this decomposition is valid only for quartic forms, which limits its use to fourth order tensors. This is not such a major limitation since modeling diffusion with a too high order tensor may overfit the signal. Future work will be done to investigate the performance of these strategies on real data sets. This work may be of great interest for registration methods based on HOT (or on spherical harmonics as mentioned in introduction) and also for the construction and use of atlases based on such models.

6. REFERENCES


5. CONCLUSIONS

In this paper, we have proposed a new reorientation strategy dedicated to the warping of HOT images. The method is based on the decomposition of an HOT using several 2nd