WAVELET-BASED VARIATIONAL DEFORMABLE REGISTRATION FOR ULTRASOUND

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ABSTRACT
The use of ultrasound as an imaging modality has many clinical applications. Robust and efficient image registration techniques are desirable in many of those applications but, because of the inherent artifacts in ultrasound, the registration problem is challenging. We propose a novel deformable registration method using a discrete wavelet transform (DWT) for ultrasound images. Energy maps were constructed from the details of the DWT in the multiresolution pyramid. The energy maps were then registered through minimizing and energy functional, using the calculus of variations, from coarser to more detailed levels of the pyramid. The energy maps were constructed from synthetic images and human liver ultrasound images. The experiments showed the advantages of the proposed method as an efficient and noise-robust registration framework.

Index Terms— Image registration, Wavelet transforms, Variational methods, Multiresolution analysis, Ultrasound imaging

1. INTRODUCTION
Medical ultrasound is a real-time, cost-effective, non-invasive imaging modality with many clinical applications. Ultrasound deformable image registration is essential for analysis or fusion of soft-tissue images. The registration is a challenging task because ultrasound has non-linear characteristics and many inherent artifacts, including speckle and a generally low signal-to-noise ratio.


There has also been recent work in using a discrete wavelet transform (DWT) [7] in medical image registration. Sharman et al. [8] proposed using the wavelet high-pass image to obtain control points to find convex hulls and principal axes using principal component analysis. Hongli et al. [9] introduced a modified scheme of DWT. The method applied thresholding intensity of the transform to extract feature space. Oubel et al. [10] used complex wavelet transform (CWT) with multilevel free form deformations. Li et al. [11] proposed constructing energy maps from the details of the discrete wavelet frame transform (DWFT) and using genetic algorithms to minimize the sum of absolute distance between images. Pauly et al. [12] used redundant discrete transform (RDWT) which removes the down-sampling operation from the DWT to construct energy map from the details of all levels in addition to the approximation of the last level.

We present a variational deformable registration method based on DWT. The method utilizes the locality of the wavelet transform within a multiresolution framework, and the calculus of variations to model the registration problem as dense displacement field problem. To the best of our knowledge, this is the first effort to combine DWT with the variational registration to perform ultrasound image registration.

2. METHODS
The deformable registration problem is finding a deformation \( \varphi(x) \), for a given vector \( x \), that maps a template image \( T(x) \) to a reference image \( R(x) \) such that \( T(\varphi(x)) \) is similar to \( R(x) \) according to a defined similarity measure. The proposed method relies on mapping image energies instead of mapping intensities. The image energy concept is adapted from signal analysis. Image energy is illustrated as the work capacity performed to deform the image to its current state, starting from a zero energy image which is a planar surface.

The method has two main steps, creating the wavelet energy maps (details in Sec. 2.2) and applying the variational registration (details in Sec. 2.3). As illustrated in Fig. 1, a multiresolution pyramid is constructed for each image using DWT. For each image, an energy map is computed for each level using the details components of the DWT coefficients. Starting with a zero displacement field, the energy map of the
The template image is deformed to the energy map of the reference image. The variational deformable registration is performed to energy maps in a top-down fashion of the wavelet pyramid. The resultant displacement field $u$ is applied to the next level of the pyramid. The final approximate solution is the displacement field of the base level of the pyramid. Both DWT and variational registration has computationally inexpensive implementations, as discussed in [13] and [14] respectively.

The wavelet transform has a better space-frequency localization in comparison to the traditional Fourier analysis. As a result, it is suitable for analyzing images where most of the features are represented by components localized in space at different scales and resolutions. In this study, we selected the discrete Meyer wavelet transform because of two main properties, orthogonality and symmetry. Other Meyer transform properties include fast convergence on frequency domain, regularity, and infinite differentiability. The classical Meyer transform is defined by the wavelet function $\psi$ and the scaling function $\phi$ (equations 1 and 2 respectively).

$$\hat{\Psi}_M(\omega) = \sin \left[ \frac{\pi}{2} v \left( \frac{3}{4\pi} |\omega| - 1 \right) \right], \quad \frac{2\pi}{3} \leq |\omega| \leq \frac{4\pi}{3},$$

$$|\omega| \notin \left[ \frac{2\pi}{3}, \frac{4\pi}{3} \right]$$

(1)

$$\hat{\Phi}_M(\omega) = \cos \left[ \frac{\pi}{2} v \left( \frac{3}{4\pi} |\omega| - 1 \right) \right], \quad \frac{2\pi}{3} \leq |\omega| \leq \frac{4\pi}{3},$$

$$|\omega| > \frac{4\pi}{3}$$

(2)

The polynomial $v(x)$ is defined as $v(x) = x^4(35 - 84x + 70x^2 - 20x^3)$ and $x \in [0, 1]$. The discrete Meyer is implemented using finite impulse response (FIR) approximation. The discrete Meyer has been shown by others to give excellent results for ultrasound features analysis (see [15]).

2.2. Wavelet Energy Map

Both the reference image $R(x)$ and the template image $T(x)$ are decomposed by DWT to the maximum number of levels. The maximum number of levels is determined by the image size and the wavelet decomposition base function. For each level, an energy map is computed using the diagonal (d), vertical (v), and horizontal (h) details components of the transform. For a level $l$ of an image $I(x)$, the energy map $E_{I_l}(x)$ is computed using the following equation

$$E_{I_l}(x) = d^2_{I_l}(x) + v^2_{I_l}(x) + h^2_{I_l}(x).$$

(3)

The energy map stresses the details at each level, providing more clear features for the registration algorithm. Also, separating the details at multiple resolutions increases the efficiency at each level.

2.3. Variational Deformable Registration

The deformable registration problem is non-convex and ill-posed. Hence, we model it as a minimization of a regularized energy functional

$$J[u] = D[u] + \alpha S[u],$$

(4)

where $D[u]$ is a difference measure, $S[u]$ is a regularizer or smoother, $\alpha$ is a weight scalar, and $u$ is the deformation field.

The difference measure is the quantification of the difference between the reference image $R(x)$ and the template image $T(x)$. The sum of square differences that we used is

$$D[R, T, u] = \int_{\Omega} (R(x) - T(x + u(x)))^2 \, dx.$$  

(5)

The regularizer is the term used to maintain the structure of the template image $T(x)$ while it is being deformed to the reference image $R(x)$. The regularizer is weighted by the scalar $\alpha$ to maintain the desired effect ratio with the similarity measure. There are many different regularizers such as elastic, fluid, and diffusion; each regularizer can be used in a class of applications depending on the required transformation properties [14]. We use the diffusion regularizer

$$S[u] = \int_{\Omega} |\nabla u_x|^2 + |\nabla u_y|^2 \, dx.$$  

(6)

Our choice of the diffusion regularizer was motivated by the smoothing properties of the displacement, rather than elastic and fluid regularizers that are physically motivated. It is also efficient and can be implemented with $O(n)$ complexity.
We use the calculus of variations to solve the minimization problem equation (4) leading to the Euler-Lagrange equation, which is the non-linear partial differential equation

$$\alpha \Delta u(x) = f(u(x)).$$

(7)

The force function \( f(u(x)) \) is defined by

$$f(u(x)) = (R(x) - T(\varphi(x)))\nabla T(\varphi(x)).$$

(8)

We approximated the Euler-Lagrange equation (7) as the linear system \( Au = f \) using a finite difference method [14]. Then we used a fixed-point iteration method to find the approximate solution of this linear system.

3. EXPERIMENTS AND RESULTS

We conducted two experiments, one on synthetic images and another on ultrasound human liver images. In both experiments, we compared our method with an intensity-based variational registration in an equivalent multiresolution framework. All methods were implemented using MATLAB 7.8.0 with Wavelet toolbox 4.4.1, and performed on Intel Core 2 Quad CPU 2 GHz with 4 GB memory.

3.1. Synthetic images

The purpose of the synthetic-image experiment was to have an illustrative ground-truth registration to study the correctness and the efficiency of the proposed method. Correctness is determined when the proposed method, using energy converges to a solution that minimizes the difference between the deformed and the reference images. Efficiency is the comparison between the processing time of both the intensity-based multiresolution variational registration and the proposed method.

We generated a synthetic image consisting of basic geometrical shapes (Fig. 2). Artificial speckle was introduced to produce the reference image because speckle is one of the more prominent artifacts in ultrasound images. A known Gaussian displacement field (with \( \sigma = 0.2 \)) was applied in all directions of the image to generate a deformed image with ground-truth deformation. This displacement field is close to natural deformation in the human liver ultrasound [5].

We applied the variational registration using both intensity and wavelet energy maps to the reference and the template images. We measured the registration for both methods using three criteria. We calculated the difference between the solution deformation field \( \omega' \) and the ground truth deformation \( \omega \). We also computed the sum of squared differences of deformed images with respect to the reference image and the processing times. Finally, we applied the method to the synthetic image 100 times and computed the mean and the standard deviation for the deformation-field differences.

As shown in Table 1, the proposed method is close to the ground-truth values of the displacement field. It also converges at a lower value of the difference measure SSD. These results show the effectiveness of utilizing the robustness of DWT to noise and the multiresolution framework to reveal more details of data. The proposed method outperformed the intensity method with average processing time of 23.0273 seconds in comparison to 62.0574 seconds – much less than half the processing time. Generating displacement fields based on energy maps proved to be efficient and fast.

![Fig. 2. Left to right: synthetic image, difference between reference and template images, difference between reference and intensity registered images, difference between reference and wavelet energy map registered images](image)

### Table 1

<table>
<thead>
<tr>
<th>Method</th>
<th>Intensity</th>
<th>Wavelet Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{\omega-\omega'} )</td>
<td>0.0069</td>
<td>0.0015</td>
</tr>
<tr>
<td>( \sigma_{\omega-\omega'} )</td>
<td>0.8144</td>
<td>0.1750</td>
</tr>
<tr>
<td>SSD</td>
<td>10.5052</td>
<td>7.2691</td>
</tr>
<tr>
<td>Time (sec.)</td>
<td>62.0574</td>
<td>23.0273</td>
</tr>
</tbody>
</table>

Table 1. Mean and standard deviation for displacement field with respect to ground-truth, convergence SSD, and computation time in synthetic images

3.2. Human liver ultrasound images

The purpose of this experiment was to test our proposed method on a real ultrasound dataset. We tested the method on human liver ultrasound images gathered from a previous experiment taken from volunteer subjects. We applied known Gaussian deformations, similar to the one applied to the synthetic image, to the ultrasound images to have a ground-truth deformation. We applied both the proposed and the intensity based registrations to 200 pairs of ultrasound images taken for 4 different subjects; see Fig. 3 for a registration example.

As shown in Table 2, the proposed method outperformed the intensity-based registration in terms of efficiency and robustness to noise. It is noticeable that the wavelet based method converged faster and with a lower value of the difference measure. It also returned a displacement field that was closer to the original field.

![Image of human liver ultrasound registration example](image)
\begin{table}
\centering
\begin{tabular}{|c|c|c|}
\hline
Method & Intensity & Wavelet Energy \\
\hline
$\mu_{\omega-\omega'}$ & 0.0233 & 0.0017 \\
$\sigma_{\omega-\omega'}$ & 0.2760 & 0.1766 \\
SSD & 29.7000 & 6.9700 \\
Time(sec.) & 18.4000 & 1.9800 \\
\hline
\end{tabular}
\caption{Mean and standard deviation for displacement field with respect to ground-truth, convergence SSD, and computation time in ultrasound human liver images.}
\end{table}

Fig. 3. From left to right, for human liver ultrasound: reference image, template image, deformation field (red) of template overlaid on the reference (white).

4. CONCLUSIONS

We proposed a novel variational deformable registration method based on discrete wavelet energy maps in a multiresolution framework, utilizing the advantages of both discrete wavelets and variational registration. The discrete wavelet transform was robust to noise and revealed more details and features of the ultrasound image because of the multiresolution processing. The variational registration allowed manipulating the ultrasound image as a field displacement, providing non-rigid deformations as illustrated. Future work may include processing data with ground-truth fiducials, and extending the method to 3D images from computed tomography or magnetic resonance imaging.

5. REFERENCES


