ACTIVE CONTOURS IN OPTICAL FLOW FIELDS
FOR IMAGE SEQUENCE SEGMENTATION

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ABSTRACT

Using variational calculus we develop an active contour model to segment an object across a number of image frames in the presence of an optical flow field. We define an energy functional that is locally minimized when the object is tracked across the entire image stack. Unlike classical snakes, image forces and regularization terms are integrated over the full set of images in the proposed model. This results in a new formulation of active contours. The method is demonstrated by segmenting the ascending aorta in a phase-contrast cine MRI dataset. Techniques to compute the required optical flow field and a “one-click” contour initialization step are suggested for this particular modality.

Index Terms—Active contours, variational calculus, optical flow estimation, segmentation, MRI

1. INTRODUCTION

Active contours have been studied intensely in image segmentation applications since their introduction in the seminal paper by Kass et al. [1]. To the best of our knowledge however, no previous work has taken the approach we suggest in this paper; derived from calculus of variations we present an active contour model which, in the presence of an optical flow field, concurrently segments an object across a number of image frames. i.e. we develop a functional that acts across the entire set of images.

Probably, the most widely used approach for active contour segmentation across a set of images is evaluating the active contour on each image sequentially. Usually a manual or semi-automatic approach is used to initialize the contour in the first image. A contour is then evolved in each frame image and regularization energy terms. In other related work the tasks of segmentation and registration have been combined in a single framework [4-5]; these references focus on finite dimensional registration, i.e. rigid and affine transformations. The method we propose handles infinite dimensional (non-affine) cases as well and thus allows segmentation of shapes, which deform over time.

2. THEORY

Consider the input data as a continuous image series $I : \mathbb{R}^{3} \rightarrow \mathbb{R}$ such that $I(x, y, t)$ denotes the image value at spatial position $(x, y)$ and time $t$. From an optical flow based registration of image $I$ compute the particle trace $\phi : \mathbb{R}^{2} \times \mathbb{R} \rightarrow \mathbb{R}^{2}$ such that

$$
\phi(p, t) = (u(p, t), v(p, t))^T
$$

denotes the spatial position at time $t$ for the particle located at spatial position $p = (x, y)$ at time $t = 0$. Define a smoothed image gradient along the particle trace, $\psi : \mathbb{R}^{2} \times \mathbb{R} \rightarrow \mathbb{R}^{2}$,

$$
\psi(p, t) = (g(\phi(p, t), t), h(\phi(p, t), t))^T
$$

with

$$
g(x, y, t) = \left( \frac{\partial(G_{\sigma} \otimes I)}{\partial x} \right)^T, \quad h(x, y, t) = \left( \frac{\partial(G_{\sigma} \otimes I)}{\partial y} \right)^T
$$

where $\otimes$ denotes 2D convolution in the $x \times y$ domain with a 2D Gaussian kernel $G_{\sigma}$ of standard deviation $\sigma$.

The goal of the segmentation problem is to find the unknown parameterized curve $x : [0; 1] \rightarrow \mathbb{R}^2$, $x(s) = (x(s), y(s))^T$ that given a registration $\phi$ denotes the desired object border across all time frames through transformation by $\phi(x(s), t)$.

We propose the following active contour for image sequence segmentation in optical flow fields. Consider the
functional $\mathcal{E}$ on $x$ describing an energy we wish to minimize:

$$\mathcal{E}(x) = \int_0^1 \mathcal{E}_{\text{image}}(x(s)) + \mathcal{E}_{\text{reg}}(x(s)) \, ds$$

with image energy

$$\mathcal{E}_{\text{image}}(x, s) = \frac{1}{2} \int_0^1 -|\psi(x(s), t)|^2 \, dt$$

and regularization energy

$$\mathcal{E}_{\text{reg}}(x, s) = \frac{1}{2} \int_0^1 \alpha|\phi(x(s), t)|^2 \, dt$$

where $\phi$ denotes the derivative of $\phi$ with respect to the curve parameter $s$, and $\alpha$ is a user defined weight. The term $\mathcal{E}_{\text{image}}$ attracts the contour to edges present in all image frames while the term $\mathcal{E}_{\text{reg}}$ regularizes the solution – again across all frames.

In the following let $\dot{x}$ and $\ddot{x}$ denote the first and second order derivatives of the curve parameterization $x(s)$. Using the notation $\mathcal{E}(x) = \int_0^1 F(s, x, \dot{x}) \, ds$, from the Euler-Lagrange equations we find necessary conditions for the functional to be minimal:

$$\frac{\partial F}{\partial x} - \frac{d}{ds} \frac{\partial F}{\partial \dot{x}} = 0, \quad \frac{\partial F}{\partial y} - \frac{d}{ds} \frac{\partial F}{\partial \dot{y}} = 0$$

We split $F(s, x, \dot{x})$ in two parts corresponding to the image and regularization energies respectively such that

$$\mathcal{E}(x) = \int_0^1 F_{\text{image}}(s, x, \dot{x}) + F_{\text{reg}}(s, x, \dot{x}) \, ds.$$  

For vector function $\phi$ denote its partial derivatives $\phi_x = \frac{\partial \phi}{\partial x}$, $\phi_y = \frac{\partial \phi}{\partial y}$, and define matrices

$$\mathbf{J}_\phi = \begin{bmatrix} \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} \end{bmatrix}, \quad \mathbf{J}_\psi = \begin{bmatrix} \frac{\partial \psi}{\partial x} & \frac{\partial \psi}{\partial y} \\ \frac{\partial \psi}{\partial x} & \frac{\partial \psi}{\partial y} \end{bmatrix}$$

$$\mathbf{H}^u = \begin{bmatrix} \frac{\partial^2 u}{\partial x^2} & \frac{\partial^2 u}{\partial y \partial x} \\ \frac{\partial^2 u}{\partial y^2} & \frac{\partial^2 u}{\partial y^2} \end{bmatrix}, \quad \mathbf{H}^v = \begin{bmatrix} \frac{\partial^2 v}{\partial x^2} & \frac{\partial^2 v}{\partial y \partial x} \\ \frac{\partial^2 v}{\partial y^2} & \frac{\partial^2 v}{\partial y^2} \end{bmatrix}$$

that (when ignoring the temporal parameter $t$) resemble the Jacobian of $\phi$ and $\psi$ and the Hessian of $u$ and $v$ respectively. For the derivation below we furthermore introduce a matrix $A$ (required only for intermediate results):

$$A = \begin{bmatrix} \frac{\partial^2 u}{\partial x^2} & \frac{\partial^2 u}{\partial y \partial x} \\ \frac{\partial^2 v}{\partial x^2} & \frac{\partial^2 v}{\partial y \partial x} \end{bmatrix}.$$ 

We then compute

$$\frac{\partial F_{\text{reg}}}{\partial x} = \alpha \int_t \left( \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial u}{\partial y} \frac{\partial u}{\partial y} \right) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y \partial x} \right) + \alpha \int_t \left( \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial v}{\partial y} \frac{\partial v}{\partial y} \right) \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y \partial x} \right) \, dt$$

and (omitting the details)

$$\frac{d}{ds} \frac{\partial F_{\text{image}}}{\partial x} = 0 \implies \frac{d}{ds} \frac{\partial F_{\text{image}}}{\partial \dot{x}} = 0$$

whereby the full Euler-Lagrange equation for $x(s)$ becomes

$$- \int_t \left( J_{\phi_x} \right)^T P_x + \left( J_{\psi} \right)^T (A \dot{x}) \, dt = 0$$

(1)

By analogy the equation for $y(s)$ becomes

$$- \int_t \left( J_{\phi_y} \right)^T P_y + \left( J_{\psi} \right)^T (A \dot{y}) \, dt = 0$$

(2)

We introduce an artificial time variable $\tau$ to solve equations (1) and (2) by numerical integration:
\[ \frac{\partial x}{\partial \tau} = \frac{\partial F}{\partial x} - \frac{d}{ds} \frac{\partial F}{\partial x}, \quad \frac{\partial y}{\partial \tau} = \frac{\partial F}{\partial y} - \frac{d}{ds} \frac{\partial F}{\partial y} \]  

(3)

As the evolution of equation (3) stabilizes \( \frac{\partial x}{\partial \tau} = 0 \) we have obtained a solution to equations (1) and (2).

The contour \( \mathbf{x}(s, \tau) \) is discretized to a finite resolution vector \( \mathbf{x} \). The functions \( \phi \) and \( \psi \) are represented at finite resolution corresponding to the resolution of the acquired discrete image \( I \).

### 3. SEGMENTATION IN PC-MRI

We demonstrate the proposed method by a “one-click” segmentation of the ascending aorta in a phase-contrast cine magnetic resonance imaging (MRI) acquisition. A frame-by-frame active contour model has previously been proposed to solve this problem [6].

#### 3.1. Acquisition

To evaluate the algorithm one dataset with 32 frames (cardiac phases) was acquired in a healthy volunteer using a Philips Intera 1.5T MRI scanner. The image series was acquired using a conventional segmented \( k \)-space (factor=3) 2D prospective ECG and diaphragm navigator gated phase contrast flow sequence with a non-isotropic spatial resolution of 1.4x1.4x5 mm\(^3\). The flip angle was 25°, TR=6.4ms, TE=3.3ms, the matrix size was 256x256 and 2 signal averages were acquired. The 2D slice was placed approximately 3 cm above the aortic valve. The scan technique used prospective triggering and the navigator was played out for 35 ms at the beginning of each cardiac cycle. The 2D image series was acquired in approximately 3 minutes with a navigator efficiency of 50%.

#### 3.2. Optical flow field estimation

The required particle trace \( \phi \) is computed from an initial multi-scale optical flow based registration [7-8]. Horn and Schunck proposed a method in which the material derivatives along the unknown displacement field were minimized, thus relying on an assumption of intensity conservation [7]. For cardiac MRI it is not generally true however that a particle’s intensity is constant throughout the cardiac cycle. We consequently use a modified algorithm suggested by Cornelius and Kanade [8]. They loosened the requirement of strict intensity conservation by letting the material derivative be an additional unknown parameter to be estimated.

#### 3.3. Active contour initialization

Inspired by previous work [9] we utilize the computed optical flow field \( \phi \) to perform a cluster analysis of the registered phase images using \( k \)-means clustering [10]. As the velocity profiles inside the vessels of interest differ from the remaining image components, the perimeters of the corresponding clusters constitute good starting points for the active contour.

#### 3.4. Implementation

**Hardware:** a Windows XP 64-bit workstation containing a 2400 MHz Intel Core 2 Duo processor, 4 GB of memory, and an Nvidia GeForce GTX 280 GPU with 1 GB memory. **Software:** all CPU code was written in C++. Compute intensive and massively data-parallel portions of the code (to be specified below) were implemented on the GPU using CUDA C (nvidia.com/cuda).

Computation of \( \phi \) and \( \psi \) were performed in parallel on the GPU. Details of the GPU implementation of the optical flow algorithm has been reported previously [11]. This resulted in a frame-to-frame cyclic registration of the MRI dataset. The registration was performed both forwards and backwards in time. The particle trace \( \phi \) was then constructed by concatenation of deformation fields using the central image as the initial frame. The \( k \)-means clustering algorithm was also implemented on the GPU. We used 7
clusters and 100 iterations from an initial cluster configuration with a mean velocity of 0 for every cluster.

The active contour model was implemented single-threaded on the CPU. We discretized $x$ into 15 points and used the settings $\alpha = 10^{-5}$, $\beta = 0.05$ for the results presented in the following section.

4. RESULTS

Fig. 1A shows the central frame (modulus image) of the image sequence (left) and the obtained $k$-means clustering result (right). By a single mouse click the user selected the ascending aorta as the target vessel for the subsequent segmentation. Fig. 1B depicts the initial contour (perimeter of the corresponding cluster) on the central, a late systolic, and a late diastolic frame respectively. The final result after evolution of the proposed active contour model from the initial configuration is shown in Fig. 1C. Sequentially, 25 time steps with $\psi$ smoothed by Gaussian kernels of standard deviation $\sigma = 5.5$, $\sigma = 1.5$, and $\sigma = 1.0$ were applied.

Initial computation of $\phi$, $\psi$, $J^\phi$, $J^\psi$, $H^u$, and $H^v$ took roughly 1 second. An additional second was spent computing the cluster map. The 75 iterations of numerical integration solving equation (3) lasted 200 ms. In other words; the result obtained in Fig. 1 was computed in a few seconds.

5. DISCUSSION

We presented a general active contour model, which in the presence of an optical flow field segments and tracks an object throughout an image sequence. Unlike classical snakes [1], image and regularization energies were derived from the entire image stack.

To exemplify the proposed technique we looked specifically at segmentation of the ascending aorta in a phase-contrast MRI acquisition with 32 frames. This is a challenging task, since the ascending aorta undergoes a substantial amount of movement and intensity variation throughout the cardiac cycle (Fig. 1C, horizontal line). Nevertheless, the proposed method was capable of quite accurate segmentation and tracking.

Significant portions of the implementation were parallelized on the GPU to keep the computational time low. Since the active contour itself was discretized into a relatively small number of points however, computation of contour evolution is not an obvious candidate for GPU acceleration. It would be straightforward however to parallelize the solution of equation (3) on a multicore CPU system if a faster implementation is desired.

Implicit function representations, i.e. level set formulations, offer a number of advantages over explicit representations in relation to e.g. contour resampling issues and handling of topological transformations [12]. For the application presented in this paper however, the explicit representation worked very well. Even so, it would be interesting future work to derive a level set formulation corresponding to the proposed minimization problem of the energy functional $E(x)$. This could probably be achieved along the lines described in ref. [12].

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7. REFERENCES