NON-UNIFORMITY CORRECTION USING COSINE FUNCTIONS BASIS AND TOTAL VARIATION CONSTRAINT

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ABSTRACT
In this paper we introduce a new non-uniformity correction technique for receiver dependent intensity fluctuations in NMR images. Our method was designed for lower limb images, particularly those acquired in the context of muscular dystrophy studies. The new approach was motivated by the fact that in pathological cases we cannot make assumptions about the characteristics of the various tissues in the image, which is a prerequisite and a main limitation for the numerous techniques proposed in the literature. In this work we considered a parametric model for the non-uniformity field based on combination of cosine functions. The estimation of the parameters was done by minimizing a cost function that reduces the variance in the subcutaneous fat as well as the total variation of the non-uniformity function. Experimental results were promising and showed the efficiency of the proposed approach.

Index Terms—Non-Uniformity, Cosine basis, Total variation,

1. INTRODUCTION
Magnetic resonance imaging (MRI) is a powerful non-invasive imaging technique with a tremendous potential for quantitative analysis of tissue properties. The accuracy of such analysis is however highly dependent on the image acquisition characteristics and can be affected by numerous hardware imperfections. In this paper we focus on receiver dependent non-uniformity that is a slow intensity variation of the same tissue over the image domain. Non-uniformity correction is of paramount importance since it impacts directly the quantitative studies performed on NMR images. Different methods have been proposed to address this problem. For instance, filtering based techniques approximate the intensity non-uniformity by low pass filtering the observed images [1, 2]. These techniques are not reliable in the context of quantitative analysis because they assume that the low frequency variation in the NMR images corresponds to non-uniformity field. Such an assumption may not be true in pathological cases. Statistical based approaches compute the non-uniformity field using a classification framework where the number of tissues is known as well as the distribution intensity inside each tissue [3, 4]. This prior knowledge is a limitation in the context of our application. In [5] a correction technique, that is free of prior knowledge about the tissue, was proposed but it assumed a Gaussian distribution for the non-uniformity field. Spatial domain based approaches aimed to estimate the non-uniformity by reducing image entropy [6, 7] or the intensity variability inside each class of tissue [8, 9]. The non-uniformity field was approximated in this case by spline functions [6] or polynomial functions [8, 7, 9]. While spline functions can provide a good approximation of the non-uniformity field, the polynomial models are not accurate. Indeed, the non-uniformity function is highly dependent on the acquisition process and the coil used as well as the image object. An exhaustive review of correction technique can be found in [10].

In this work, we were interested in correcting non-uniformity in NMR images of lower limbs (leg and thigh), with longer term goal, to investigate a variety of neuromuscular disorders. In this context, making assumptions about the muscle intensity distribution is not straightforward because a pathological muscle may display fatty infiltration, inflammation and/or fibrosis. Furthermore, for Dixon images [11] the ratio between the fatty component pixel intensities in muscles and the mean value in the subcutaneous fat provides an estimate of fatty infiltration progression. If we use a correction method based on entropy minimization, this ratio will be artefactually increased. This is explained by the fact that fat infiltrated pixels, lying in the muscle region, have an intensity range close to the subcutaneous fat and are represented by the same mode in the histogram. In this case, minimizing the entropy amounts to reducing the variability of all pixels that correspond to fat (and potentially increase the intensity of muscle pixel that are fat infiltrated). Such a processing will have an impact on the quantification of fat infiltration. For this reason we did not consider entropy minimization based techniques. To estimate the non-uniformity field, we can rely only on the assumption of the uniformity of the signal in the subcutaneous fat. We need then to estimate the non-uniformity values in the region of the muscle. An interpolation technique like spline or polynomial would not be efficient because the muscle groups occupy a very large fraction of the limb segment. Parametric models are an attractive solution to the problem. However, considering a polynomial representation of the non-uniformity field is not very accurate.

In this paper we introduce a new technique to estimate the non-uniformity field based on the homogeneity assumption in a limited domain of the image. Based on the observation that the non-uniformity field is a low frequency signal, we express it as a finite sum of discrete cosine functions. The estimation of the parameter of the non-uniformity field is done through the minimization of a convex cost function. A detailed presentation of the method is provided in the second section. Next, we will focus on the experimental validation of the technique, from which conclusions will be drawn in the fourth section.

2. NON-UNIFORMITY ESTIMATION TECHNIQUE
A convenient way of designing a correction method consists in modeling the intensity non-uniformity as resulting from a smooth multiplicative field. Hence we can write:

\[ I(v) = U(v)B(v) + n(v) \quad \text{For each } v \in \Omega \] (1)
Where \( v = (x, y, z) \) refers to a voxel location, \( I \) is the observed NMR image, \( U \) is the uncorrupted image, \( B \) is the non-uniformity field and \( n \) is the noise.

Knowing that the non-uniformity field is a smooth function, we can approximate it by a finite sum of cosine discrete functions. The choice of the cosine functions is motivated by the fact they represent a discrete orthogonal basis. Hence we can define the non-uniformity field as the following

\[
B(v) = \sum_{k_x=0}^{n_x-1} \sum_{k_y=0}^{n_y-1} \sum_{k_z=0}^{n_z-1} h_{k_x,k_y,k_z} C_{k_x}(x) C_{k_y}(y) C_{k_z}(z) \tag{2}
\]

\[
C_{k_d}(d) = \cos \left( \frac{\pi(2d+1)k_d}{2L_d} \right) \quad \text{with} \quad d \in \{x, y, z\} \tag{3}
\]

The estimation of the non-uniformity field amounts to computing the coefficients of the cosine functions that will be represented by the vector \( h = (h_{0,0,0}, h_{1,0,0}, \ldots, h_{n_x,n_y,n_z}) \). Under the assumption that the signal in the subcutaneous fat is uniform and that it can be approximated by its mean value (called \( \mu_f \)), the estimation can be performed by minimizing the following quadratic cost function [12]:

\[
E(h) = \sum_{v \in \Omega_{sf}} ||I(v) - \mu_f B(v)||^2 \tag{4}
\]

Where \( \Omega_{sf} \) is the image domain relative to the subcutaneous fat.

The efficiency of such an approach is highly dependent on the domain \( \Omega_{sf} \). The non-homogeneity field can be reconstructed if the observed samples are uniformly distributed in the image domain. This is unfortunately not the case of our application where the hypothesis of signal homogeneity is valid only for subcutaneous fat. The system obtained using only pixels of subcutaneous fat area is bad conditioned because the restriction of the cosine function to this domain is no longer orthogonal. We can observe very important oscillations in the region that were not sampled. To overcome this problem we considered an additional constraint that aims to reduce the oscillations of the non-uniformity field. Such a constraint can be modeled by the total variation of this function. Thus, we can estimate the cosine function coefficient by minimizing the following cost function:

\[
E(h) = \sum_{v \in \Omega_{sf}} ||I(v) - \mu_f B(v)||^2 + \lambda \sum_{v \in \Omega} ||\nabla B||^2 \tag{5}
\]

The first term of the cost function ensures the homogeneity of the subcutaneous fat signal where the only variation source is the non-uniformity field. The second term is the total variation of the field in the entire domain. Its role is to reduce the oscillations in the resulting function. \( \lambda \) is a parameter that defines the trade-off between the two constraints. By introducing the square of \( L^2 \) norm of the non-uniformity gradient, we still have a quadratic cost function with respect to the variable \( h \). The optimal value of \( h \) can be obtained using a gradient conjugate approach. The derivative with respect to \( h \) is expressed as the following:

\[
\frac{\partial E}{\partial h_{k_x,k_y,k_z}} = -2\mu_f h_{k_x,k_y,k_z} \sum_{v \in \Omega_{sf}} C_{k_x,k_y,k_z}(v) (I(v) - \mu_f B(v)) + 2\lambda \sum_{v \in \Omega} \left( \frac{\partial C_{k_x,k_y,k_z}}{\partial x} + \frac{\partial C_{k_x,k_y,k_z}}{\partial y} + \frac{\partial C_{k_x,k_y,k_z}}{\partial z} \right) \tag{6}
\]

It is important to point out that we are minimizing a quadratic cost function which can deal with Gaussian noise. In our case the signal in the fat is important and the noise distribution can be approximated by a Gaussian distribution. But we can improve the robustness of the estimation using a trimmed least square approach.

### 3. EXPERIMENTAL VALIDATION

In this section we will evaluate the performance of the proposed algorithm. First, we will show the impact of the total variation constraint on the estimation quality as well as the influence of the parameters selection. Second we will compare quantitatively the performance of our algorithm to the one presented in [6]. Before discussing the performance of our algorithm, we have to point out that to implement our method one has first to determine the region that corresponds to the subcutaneous fat. This region is extracted by a simple thresholding because it is well contrasted with respect to the other tissues in the muscle.

#### 3.1. Impact of the total variation constraint

In a first step we considered phantom data because such a simple data set will provide us with a good idea about the performance of the proposed technique. The data set that we used corresponds to 2D acquisition of a water phantom using a 3T Siemens scanner. The experiment that we designed is the following: we define the domain \( \Omega_{sf} \) as shown in [Fig.1] (the equivalent of a subcutaneous fat in a calf for example). This region will be used in equation (4) and (5) to estimate the optimal parameters of the non-uniformity function. For a quantitative comparison between the two approaches, we considered the coefficient of variation as a criterion. The coefficient of variation in a region is the ratio between the standard deviation and the mean of the intensity inside this region. For the phantom data we considered two regions for the evaluation the peripheral region and the region in the center [Fig.1]. We also studied the impact of the number of the cosine functions on the estimation quality. Based on the results reported in [Tab.(1)-Up], we made the following observations: (i) the two compared approaches were able to approximate the non-uniformity field based on observations that are local and not uniformly distributed in the image domain. For instance, the coefficient of variation was not only reduced in the region considered in the optimization but also in the internal region of the phantom (ii) when considering only the peripheral region involved in the computation of coefficient we can notice that better variability reduction is obtained with higher number of cosine function. This is a natural outcome because a better approximation of any function is insured by higher number of basis function. However we noticed that with the first formulation and when \( k = 7 \), the central region of the phantom presented many artifacts because of important oscillations of the computed non-uniformity field in this area. This problem was resolved when we added the total variation minimization constraint. To further illustrate the impact of the constraint on real data, we considered a 2D slice of the thigh acquired with the same parameters as the phantom. The results are shown in [Tab.(1)-Down]. This example clearly showed that for the first formulation, a good correction was only limited to the subcutaneous fat area and the image quality was lower inside the muscle. By minimizing the total variation of the field, the oscillations were reduced which ensured a better correction. In [Fig.2] we can see the resulting images after correction which confirms the observations made based on the quantitative evaluation.
### 3.2. Comparison with entropy minimization based technique

To further assess the performance of the proposed technique, we compared it to the one presented in [6]. This method consists in correcting the image by minimizing its entropy. The field is modeled using spline interpolation. This algorithm is available in the BrainVisa software \(^1\). For comparison we considered a dataset of eight Dixon volumes acquired by a 3T Siemens scanner (for each subject we have one volume for water and one volume for fat [Fig.3]). The volumes correspond to the thigh of 3 healthy subjects and one phantom. The size of each volume is \((330\times160\times160)\) voxels. Regarding our technique, we estimated the non-uniformity field on fat NMR volumes and applied then this correction to the water NMR volumes. For technique in [6], we corrected independently the water and the fat volumes. We made this choice because for healthy patient the signal in the region of the muscle groups is very low in the NMR fat volumes (due to the absence of fat) and thus it was not easy to correct this area based on fat image only using entropy minimizing. For evaluation we considered the coefficient of variation in the subcutaneous fat as well as the muscle tissue. We made comparison using the optimal parameters for each algorithm. For our algorithm we set \(\lambda = 1000\) and \(n_{kx} = n_{ky} = n_{kz} = 5\). Regarding the BrainVisa software, we optimized the \(K_r\) parameter. We considered different values ranging from 20 to 0.5 and we performed the correction of the different data set. We selected then the value of \(K_r = 1\) that gives the best coefficient of variation. Evaluation in the muscle region was possible because we used a data set composed of healthy subjects were the muscle tissue is homogeneous. It is important to point out, that for a pathological case, evaluation using coefficient of variation is not feasible because the muscle tissue is no longer uniform. In [Tab. (2)] we reported the coefficients of variation for each tissue before and after correction using the two algorithms. We used the fat NMR images to evaluate the coefficient of variation of the signal in the subcutaneous fat and the water images to compute the coefficient of variation corresponding to the muscle. We can notice that our algorithm outperforms the one based on entropy correction in the region subcutaneous fat and has similar performance for muscle regions. In [Fig.3] we showed some examples of correction using both algorithms. We can see that our algorithm achieves better correction for the fatty region and the bone.

Regarding pathological cases, Results presented in [Fig.4], showed that with entropy minimization based technique, the homogeneity inside the muscle was not completely removed. This is explained by the fact that we provide our algorithm with a mask that defines the region to be homogenized. Indeed, for high field MR scanner, using only the histogram information may not be reliable. In figure [Fig.5], we noticed that entropy minimization technique overestimated the intensity of fat in pixels belonging to muscle area (see region pointed by an arrow). Indeed, the entropy minimization based approach aims to homogenize pixels that belong to the same intensity range regardless their spatial position. In the presented case it increased the intensity of fat infiltrated pixels belonging to the muscle area in order to reduce variability of the histogram mode presenting the fat. Using our method, we are just interpolating the non-uniformity field inside the muscle region.

### 4. CONCLUSION

In this paper we presented a technique of receiver dependent non-uniformity correction applied to NMR volumes of the lower limb. To achieve this correction, we assumed that the signal is uniform in one given tissue that is segmented using a simple image thresholding. This approach implies that B1 transmit heterogeneities of the images are either non-existent (which is the case of the data we used) or have been compensated for. To model the receiver field,
we adopted a parametric model using cosine functions combination. Our main contribution is the addition of the total variation constraint in the estimation of the function parameters. The experimental results showed that our method was able to perform good correction in spite of the lack of information about the non-uniformity function inside a large region. Future direction to improve this work is to use a more robust estimator than the quadratic error. This might be achieved using trimmed least square which has the advantage of conserving the convexity of the problem. To improve correction inside thigh or leg muscles, we could also perform a learning step on healthy subjects to add additional constraints when inferring the non-uniformities for pathological cases.

5. REFERENCES


