ON THE INFLUENCE OF INTERPOLATION ON PROBABILISTIC MODELS FOR ULTRASONIC IMAGES

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ABSTRACT

The influence of the cartesian interpolation of ultrasound data over the final image statistical model is studied. When fully formed speckle is considered and no compression of the data is done, we show that the interpolated final image can be modeled following a Gamma distribution, which is a good approximation for the weighted sum of Rayleigh variables. The importance of taking into account the interpolation stage to statistically model ultrasound images is pointed out. The interpolation model here proposed can be easily extended to more complex distributions.

Index Terms—Speckle probability density, ultrasound interpolation, ultrasound speckle, acoustical properties.

1. INTRODUCTION

Some image filtering and segmentation techniques for ultrasound imaging, as those approaches based on maximum likelihood and maximum a posteriori [1], rely on an accurate statistical model for the different regions in the image. This model is usually derived from the analysis of the acoustic physics and the information available of the ultrasound probe. However, the whole information during the acquisition process is not usually available, and therefore some suppositions must be considered. For example, images provided by practitioners usually do not include the acquisition parameters as gain and/or contrast adjustment. Additionally, some of the steps of the acquisition process may be unknown, depending on the commercial firm of the ultrasound equipment.

When estimating probability density functions (PDFs) for filtering or segmentation, a common way to deal with the lack of information is to use empirical approximations which accurately fit these PDFs to the speckle patterns. This methodology has been used in literature [1, 2] for different kind of distributions. It offers an empirical methodology to test the goodness of fit of the distributions to real data in the last step of the acquisition process, avoiding this way, the problem of propagating the probabilistic model through the whole acquisition process.

Speckle in ultrasound image can be seen as a random process whose statistical features provide information about the tissue. Existence of deterministic component in this process depends on the number of obstacles (scatters) into the resolution cell and their size in comparison with the wavelength of ultrasound signal. Depending on the scatter number density per cell (SND), four types of speckle can be defined: (1) Fully formed: large number of scatters and non-existence of deterministic component, modeled by Rayleigh distribution. (2) Fully resolved: large number of scatters and existence of deterministic component, modeled by Rice distribution. (3) Partially formed: non-large number of scatters and non-existence of deterministic component, modeled by K distribution. (4) Partially resolved: non-large number of scatters and existence of deterministic component, modeled by K-Homodyne distribution.

In this paper we will focus on the first model, i.e. fully formed speckle. In fully formed speckle regions [3], acquired signal can be modeled following a Rayleigh distribution. However, to form the final Cartesian image, these Rayleigh distributed data have to be interpolated. Thus, the resulting image will no longer follow a Rayleigh distribution. Our aim will be to model this final distribution taking into account the interpolation process. Albeit being the simplest of the proposed models, the initial Rayleigh distribution considered can be found in several practical situations, for example in ultrasound imaging of blood. Blood cells behave like tiny randomly distributed scatters, so blood speckle can be classified into fully formed speckle.

If no compression of the data is done (which will be an assumption throughout the paper), we show that the some results provided in literature [1, 2] hold with the interpolation probabilistic model here presented and that this approach can be extended easily to other distributions. We present an exhaustive statistical test using real-life cases for cardiac ultrasound images which will confirm that the interpolation model accurately fits the data.

The paper is structured as follows: In section 2 the interpolating transformation is studied and a distribution is proposed for approaching the theoretical distribution. In section 3 the proposed distribution is tested and compared with other...
works of the literature. In section 4 the interpolation model is used for classifying tissues for real and simulated images. Finally, in section 5 we conclude analyzing the results.

2. INTERPOLATION MODEL

Ultrasonic images are constructed from a number of acoustic “lines” or vectors usually organized in a sequential pattern [4]. These vectors form lines in the image after conversion by envelope detection. Each line represents a time record of the scattered waves from different depths. The process of image formation begins with a pulse packet emission which travels along the beam vector axis and changes shape according to characteristics of the media. The traveling pulse is scattered by objects placed at a scattering depths and cause delays in the pulse. Reflections are received by the transducer and, considering a constant sound speed, the depths of the scattering objects can be estimated. These intercepted waves are integrated over the surface of the transducer with a suitable weighting and time delays are added for focusing and beam-forming. The amplitude of the envelope record is usually logarithmically compressed but this is optional depending on the ultrasonic machine. At this point, when fully formed speckle regions are observed, a Rayleigh probabilistic distribution is often considered [5]. After this step all the lines are interpolated to form a complete Cartesian image from a number of image lines arranged in their geometrical attitude [4].

In this section we discuss the influence of interpolation on the probabilistic model when fully formed speckle regions are considered. Although this strategy can be extended to other distributions, we will pay special attention to the Rayleigh case.

Let \{X_i\} be independent identically distributed (IID) random variables (RVs) which follow a Rayleigh distribution \(f_X(x)\):

\[
f(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right), \quad x \geq 0
\]

When a simple scheme of interpolation is considered such the bilinear one in the 2D-case or trilinear in the 3D-case (which is likely to be the one used by the ultrasound machine because of its computational efficiency), the resultant interpolated value of the pixel can be calculated as:

\[
Y = \sum_{i=1}^{n} w_i X_i, \quad \text{where} \quad \sum_{i=1}^{n} w_i = 1
\]

The resultant PDF of the interpolated RV has no closed expression and several ways for calculating it has been presented in the last years [6]. In this work, we will consider a numerical approach based on quadrature methods due to its simple implementation, since we want to study the behavior of this PDF in order to validate the empirical approaches of the literature.

One simple way to calculate \(F_Y(y)\) is to see it as the convolution of the weighted PDFs of independent RV. This way, a closed expression can be obtained when characteristic functions are used since a Rayleigh distribution admits a characteristic function which is known, though not simple [6]:

\[
\phi_X(t) = E\{e^{tX}\} = 1 + i\sigma te^{-\sigma^2 t^2/2} \sqrt{\pi/2} \left( e^{t^2/2} + 1 \right) = 1 F_1 \left( 1, \frac{1}{2}, -\frac{\sigma^2 t^2}{2} \right) + i\sqrt{\pi/2} t \sigma e^{-t^2 \sigma^2/2}.
\]

So, the characteristic function of \(Y\) is

\[
\phi_Y(t) = \prod_{i=1}^{n} \phi_i(t),
\]

where \(\phi_i(t)\) is the characteristic function of each \(w_i X_i\). Note that \(w_i\) affects to \(X_i\) in such a way that \(Y\) can be considered as the sum of Rayleigh RVs with different \(\sigma\). The PDF is obtained from Eq. (4) by numerical quadrature.

This distribution is not practical to be used in statistical estimation of real data, due to the large number of parameters to estimate. A simplified model must be used. In this paper a Gamma distribution to approximate the exact distribution of the sum of Rayleighs is considered. Note that a Gamma PDF has only two free parameters and the behavior of the tails is similar to the PDF of \(Y\). In addition, in literature Gamma PDFs has also been used to model this kind of speckle [1], but no justification has been given.

In Fig. 1 we show the characteristic function of a Gamma \(\phi_Y\), Gaussian \(\phi_G\) and theoretical sum of Rayleigh \(\phi\) for 4 terms and the error committed in the approximation for an increasing number of terms.

In this figure we can see that the characteristic function of a Gamma distribution offers a better behavior than the Normal distribution even when the Central Limit theorem can be applied. Experiments were made considering the same weights of the RVs, however this approach can be done for arbitrary weights and the result still holds (See Fig. 1.c).

In order to test this assumption we simulate Speckle based in the acquisition model in the same fashion as it is done in [7]. This method scans an image and records the data in a matrix which is corrupted by means of the speckle formation model of [3] where the tissue is modeled as a collection of scatters that are so numerous and of size comparable to the wavelength. The speckle pattern is obtained by means of random walk which does not assume any statistical distribution in order to avoid any bias of the results.

In Fig. 2 we show the reconstructed image when no coherent echoes exist and the number of scatters is high enough to consider the speckle as a fully formed speckle pattern. As it is shown, the histogram of the image before interpolation follows a Rayleigh distribution whereas, in the case of the interpolated image, the result shows clearly that a Gamma distribution accurately fits to the data histogram.

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Fig. 1. Characteristic functions for Gamma, Normal and Interpolated Rayleighs RVs. (a) 4 terms characteristic functions and the error committed (dash-dotted lines). (b) Error of the approach of Gamma and Normal when number of terms increases. (c) Error of the approach of Gamma and Normal when number of terms increases for random weights for each case.

Fig. 2. (a) Fully formed speckle pattern. (b) Histogram of the received signal and Rayleigh distribution estimate. (c) Histogram of the reconstructed image and Gamma distribution estimate.

Fig. 3. Acceptance rate of the goodness-to-fit test for each distribution.

3. GOODNESS OF FIT TEST

In this section we study the goodness of fit test in the same way as was done in [1], this test provides the best probability distribution which fits to real data. The study is extended to a representative set of distributions that were proposed in the literature: Gamma, Log-Normal, Rayleigh, Normal, Nakagami, Beta, Rician Inverse Gaussian (RIIG), Rice, Exponential and K. We will show that the Gamma is the distribution that best fits the real data assuming the interpolation model.

The images of the data bank were obtained from a clinical machine Philips Medical Systems iE33 with the software PMS5.1 Ultrasound iE33 4.0.1.357 taken to real patients. In this work we have 120 images of size 1024 x 768 and 8 bits.

In order to obtain a proper statistical model, it is necessary to assume spatial independency between pixels. However, the independency assumption does not usually hold. In order to avoid spatial correlation the image is subsampled by a factor of 6.

A \( \chi^2 \) Goodness-to-fit test was done for all the images for an \( \alpha = 0.05 \) for a representative set of aforementioned distributions in areas with fully formed speckle previously segmented. Results obtained are shown in Fig. 3 where a better performance of the Gamma distribution is evident. This result holds with that one obtained in [1] for Gamma distributions and suggest that interpolation of fully formed speckle can be estimated by a Gamma as it was shown before. To confirm this hypothesis we decide to use the \( \chi^2 \) test for weighted sums of independent Rayleighs for different number of samples and weights. For this test, a bilinear interpolation is considered since real images were obtained from a 2D echograph. Weights were chosen to guarantee the contribution of the neighborhood pixels which is the most common situation in the interpolation.

Fig. 4 shows the probability of passing the goodness-of-fit test for a Gamma in 200 independent experiments of simulated images for an increasing number of samples. This figure shows that the Gamma distribution has a good behavior for fitting data distribution even for a big number of samples. Segmented areas of the real images (left ventricle) has usually about 500 samples so, for this number of samples, the probability of acceptance Gamma distributions achieved for real and synthetic experiments shows that the interpolation model can be fitted with a Gamma and confirms that interpolation should be taken into account for tissue probabilistic estimation.
Fig. 4. Probability of passing the goodness-of-fit test for a Gamma in 200 independent experiments for an increasing number of samples.

4. TISSUE CLASSIFICATION

In this section we present some results for real and simulated images in order to distinguish between blood and cardiac tissue. For this purpose we will consider a Gamma distribution as a good approach for fully formed speckle for the interpolated image as we saw in previous sections. A normal distribution can be taken for cardiac tissue, this supposition is reasonable when we consider the weighted sum of a Rice or K distribution, though a Gamma distribution is also reasonable for pseudo-Rayleigh speckle. In these cases, the PDF is more symmetric and a Normal provides a good approach. Fig. 5 shows a classification by means of distribution fitting for real and simulated images. As we can see, in both cases a Gamma distribution accurately fits the contribution of fully formed speckle.

5. CONCLUSIONS

In this work we analyze the influence of the interpolation in the probabilistic model of ultrasonic images for the case of fully formed speckle. We show that a Gamma distribution accurately fits to this model. Although the goodness-of-fit of the Gamma was empirically suggested by other authors, in this paper we make clear that this model arises from the interpolation of Rayleigh data. Additionally, we present some synthetic and real results that show that interpolated speckle can be modeled by the same distributions which approximate the theoretical distributions of the interpolated models. These results point out that a deeper study should be done on the interpolation step of the acquisition process of ultrasonic data.

6. REFERENCES